PG – 258

III Semester M.Sc. Examination, January 2018
(CBCS Scheme)
MATHEMATICS
M302T : Mathematical Methods

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer **any five full** questions.*
*2) **All** questions carry **equal** marks.*

Answer **any five full** questions.

1. a) Convert the following Initial Value problem into an integral equation $y'' + y = x,\ y(0) = 0,\ y(1) = 0$. **7**

b) Solve the following integral equation using the method of degenerate kernel

$$u(x) = \lambda \int_0^{2\pi} \cos(x+t)\, u(t)\, dt .$$ **7**

2. a) Solve $g(x) = e^x - \int_0^x e^{x-t} g(t)\, dt$ by Laplace transform method. **7**

b) Find the Neumann series solution and resolvent Kernel for the integral

$$g(x) = (1-x) + \int_0^x (x-t)\, g(t)\, dt .$$ **7**

3. a) Determine an asymptotic expansion for the integral of the form

$$I(x) = \int_0^x t^{-\frac{1}{2}} e^{-t} dt \text{ as } x \to \infty .$$ **7**

b) Use Laplace method to obtain the asymptotic expansion of

$$\int_0^{\frac{\pi}{2}} e^{-x\tan t} dt \text{ as } x \to \infty .$$ **7**

4. a) Evaluate the following using Watson's lemma $I(x)=\int_0^5 \frac{e^{-xt}}{1+t^2}dt$. **7**

b) Find the leading order term of $I(x)=\int_0^\infty \cos(xt^2-t)dt$ using stationary phase method. **7**

5. a) Solve $\frac{dy}{dx}=x+y^2;\ y(0)=1$ by Runge-Kutta method of four-slopes. Obtain $y(0.1)$ by taking $\Delta x=0.05$. **7**

b) Solve by classical Runge-Kutta explicit method of two-slopes :

$$\frac{dy}{dx}=x-y;\ y(0)=0,$$
$$\frac{dz}{dx}=y-x;\ z(0)=1,$$

Choose $\Delta x=0.05$ and obtain y(0.05). **7**

6. Derive any multi-step (Predictor-Corrector) method of Adam to find a numerical solution of $\frac{dy}{dx}=f(x,y(x));\quad y(x_0)=y_0$. **14**

7. a) Given the IBVP

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2};\ 0\le x\le 1;\ \ t\ge 0,$$

Subject to $u(x, 0) = x(1 - x),\ 0 \le x \le 1,$

$$u(0, t) = 0,$$

$$u(1, t) = 0,\ t \ge 0,$$

Obtain the solution by Schmidt finite-difference method.

Choose $\Delta x=\frac{1}{4}$ and $\Delta t=\frac{1}{64}$ and find $u\left(\frac{1}{4},\frac{1}{64}\right), u\left(\frac{1}{2},\frac{1}{64}\right)$ and $u\left(\frac{3}{4},\frac{1}{64}\right)$. **7**

b) Show that the Schmidt finite difference method is conditionally stable. **7**

8. a) Using the explicit finite-difference method find an approximate solution of the one-dimensional wave equation **7**

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2};\ 0 \le x \le 1;\ t \ge 0,$$

subject to $u(x, 0) = x(1 - x)$

$$\frac{\partial u}{\partial t}(x,0) = 0,\ 0 \le x \le 1$$

$$u(0, t) = 0,$$

$$u(1, t) = 0,\ t \ge 0.$$

Choose $\Delta x = \frac{1}{4}$ and $\Delta t = \frac{1}{64}$. Obtain the solution at first-time level.

b) Derive the finite difference approximation for second order spatial derivative in the case of a non-rectangular region. **7**